Problem 1: Show that for any simple graph G (not necessarily connected) having n vertices and m edges, if m ≥ n, then G contains a cycle.

🡪 A connected acyclic graph will have m = n – 1 edges

🡪 An acyclic graph should have m < n – 1 edges

🡪 Thus, if a graph (not necessary connected) having m = n edges, then it contains cycle

Problem 2: Suppose G = (V, E) is a connected simple graph.

Suppose S = (VS, ES) and T = (VT, ET) are subtrees of G with no vertices in common (in other words, VS and VT are disjoint).

Show that for any edge (x,y) in E for which x is in VS and y is in VT, the subgraph obtained by forming the union of S, T and the edge (x,y) (namely, U = (VS U VT, ES U ET U {(x,y)})) is also a tree.

🡪 mS = nS – 1, mT = nT – 1

🡪 mU = nS – 1 + nT – 1 + 1 = nS + nT – 1 = mU – 1

🡪 mU is a connected acycle graph (a tree)

Problem 5: Prove that if T is a tree with at least two vertices, T has at least two vertices having degree 1.

Hint. Let v be any vertex in T and think of T as a rooted tree with vertex v. Create the usual levels for the tree. Then use properties of such a tree to solve the problem.